S3 Specimen (IAL) (MA)

Q1) $H_0: \mu = 80$ $\quad n = 100$ $\quad$ critical-value : ± 1.6449

$\bar{x} = 83$ $\quad$ (5%, 1-tail)

$H_1: \mu > 80$ $\quad \sigma = 15$

$$\text{Test Stat} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{83 - (80)}{\frac{15}{\sqrt{100}}} = 2.00$$

$2.00 > 1.6449$

$\therefore$ Result is significant (reject H_0)

Evidence suggests that the director's claim is correct.

Q2a) $P \sim N(90, 9)$

$J \sim N(91, 12)$

$P(\text{required}) = P(J < P) = P(J - P < 0)$

$$\left.\begin{aligned} E(J-P) &= 91 - 90 = 1 \\ Var(J-P) &= 12 + 9 = 21 \end{aligned}\right\} (J-P) \sim N(1, 21)$$

$\therefore P(J - P < 0) = P\left(Z < \frac{0-1}{\sqrt{21}}\right)$

$= P(Z < -0.22) = 1 - P(Z < 0.22)$

$= 1 - 0.5871 = \boxed{0.4129}$

2 minutes

b) $P(\text{required}) = P(P_1 + \dots + P_{60} + 120 < J_1 + \dots + J_{60})$

$= P(P_1 + \dots + P_{60} + 120 - (J_1 + \dots + J_{60}) < 0)$

let $A = P_1 + \dots + P_{60} + 120 - (J_1 + \dots + J_{60})$,

$E(A) = 60(90) + 120 - (91)(60) = 60$ //

$Var(A) = 60(9) + 60(12) = 1260$ //

(remember $Var(aX+b) = a^2 Var(X)$)

<u>note</u>: $P_1 + \dots + P_{60} \neq 60P$ so do not square the 60 when calculating variance of A

$\therefore A \sim N(60, 1260)$

$P(\text{required}) = P(A < 0) = P\left(Z < \frac{-60}{\sqrt{1260}}\right)$

$= P(Z < -1.69) = 1 - P(Z < 1.69)$

$= 1 - 0.9545 = \boxed{0.0455}$

actual width $= w$.

Q3a) $X \sim N(w, 0.5^2)$ where $X =$ measured width

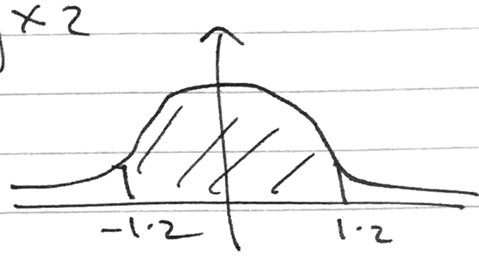
$P(\text{required}) = P(w-0.6 < X < w+0.6)$

$\Rightarrow P\left(\frac{w-0.6-w}{0.5} < Z < \frac{w+0.6-w}{0.5}\right)$

$\Rightarrow P(-1.2 < Z < 1.2)$

$\Rightarrow \left[P(Z<1.2) - 0.5\right] \times 2$

$\Rightarrow \left[0.8849 - 0.5\right] \times 2$



$\Rightarrow \boxed{0.7698}$

b) so 16 samples of X are taken.

By central limit theorem... $\bar{X} \sim N\left(w, \frac{0.5^2}{16}\right)$

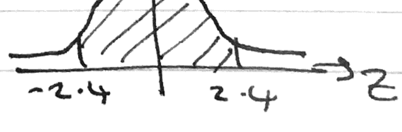
$P(\text{required}) = P(w-0.3 < \bar{X} < w+0.3)$

$= P\left(\frac{w-0.3-w}{\frac{0.5}{4}} < Z < \frac{w+0.3-w}{\frac{0.5}{4}}\right)$

$= P(-2.4 < Z < 2.4)$

$= \left[P(Z<2.4) - 0.5\right] \times 2 = 2\left[0.9918 - 0.5\right]$

$= \boxed{0.9836}$



(1% each tail, c.v = 2.3263)

c) 98% C.I $\Rightarrow \left[\bar{x} \pm 2.3263\left(\frac{\sigma}{\sqrt{n}}\right)\right]$

$\Rightarrow \left[35.6 \pm 2.3263\left(\frac{0.5}{\sqrt{16}}\right)\right]$

$\Rightarrow [35.3 \ , \ 35.9]$

(Q4a)

Position	Distance b	Depth s	R_b	R_s	d	d^2
A	100	60	1	1	0	0
B	200	75	2	2	0	0
C	300	85	3	4	1	1
D	400	76	4	3	1	1
E	500	110	5	6	1	1
F	600	120	6	7	1	1
G	700	104	7	5	2	4
						8

$\sum d^2 = 8 \quad \therefore \ r_s = 1 - \frac{6(8)}{7(48)}$

$= \boxed{0.857}$

b) $H_0: \rho = 0$
$H_1: \rho > 0$

critical value: ± 0.8929
(1%, 1-tail)

$0.857 < 0.8929$

depth increases as distance increases.
this is the claim made.

Result is insignificant.
Accept H_0.
Insufficient evidence to support the researcher's claim.

(Q5) H_0: Relative state of finances is independent of income range (no association).

H_1: Relative state of finances is not independent of income range (association exists).

$$\text{Expected no.} = \frac{\text{Row Total} \times \text{Column Total}}{\text{Grand Total}}$$

EXPECTED:

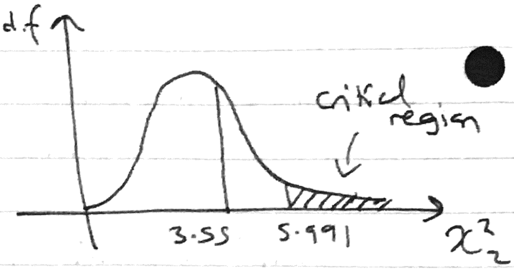
	Worse	Same	Better	
Under £15k	10.54	10.54	12.92	34
≥ £15k	20.46	20.46	25.08	66
	31	31	38	$\boxed{100}$

O_i	E_i	$\frac{(O-E)^2}{E}$
14	10.54	1.1358
11	10.54	0.0200
9	12.92	1.1893
17	20.46	0.5851
20	20.46	0.0103
29	25.08	0.6126
		3.5531..

$$X^2 = \sum \frac{(O-E)^2}{E} = 3.55$$

$$\nu = (\text{rows} - 1)(\text{columns} - 1) = (2-1)(3-1) = 2$$

$$\therefore \text{critical value} = \chi^2_2 (5\%) = 5.991$$



$3.55 < 5.991$

$\therefore$ Result is insignificant.
Accept H_0.
Evidence suggests state of finances is independent of income.

Q6) H_0: C.U.D is a good fit for these data.
H_1: C.U.D is not a good fit for these data.

Expected no. of items in each cell $= \frac{b-a}{n} \times 228$

where n = total range (12).
b = upper class limit
a = lower class limit

so.... [eg for 9-12 class, $E_i = \frac{12-9}{12} \times 228 = 57$]

Distance	0-1	1-2	2-4	4-6	6-9	9-12
O_i	22	15	44	37	52	58
E_i	19	19	38	38	57	57
$\frac{(O-E)^2}{E}$	$\frac{9}{19}$	$\frac{16}{19}$	$\frac{18}{19}$	$\frac{1}{38}$	$\frac{25}{57}$	$\frac{1}{57}$

Test Stat $= X^2 = \sum \frac{(O-E)^2}{E} = 2.75$

ν = no. of cells $- 1$ = $6 - 1 = 5$
(after pooling)

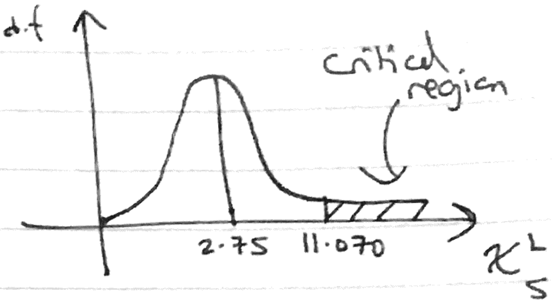
$\therefore$ critical value $= \chi^2_5(5\%) = 11.070$

$2.75 < 11.070$

$\therefore$ Result is insignificant.

Accept H_0

Evidence suggests that C.U.D is a good fit for these data.



(Q7a) Label full-time staff 1 – 6000

Label part-time staff 1 – 4000 . . . .

No. of full-time staff needed $= \frac{6000}{10000} \times 200 = 120$

No. of full part-time staff needed $= \frac{4000}{10000} \times 200 = 80$

. . . . Use random numbers to pick 120 full-time staff and 80 part-time staff.

b) Stratified ensures that each group is fairly represented -> reflects population structure.

c) $H_0: \mu_F = \mu_P$ F indicates full-time

$H_1: \mu_F \neq \mu_P$ P indicates part-time.

critical value: 2.5758

(2-tail, 1%)

$$\text{Test Stat} = \frac{\bar{X} - \bar{Y} - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}}$$

$$= \frac{52 - 50}{\sqrt{\frac{21}{80} + \frac{19}{80}}} = 2.828\ldots$$

$2.828 > 2.5758$

$\therefore$ Result is significant.
Reject H_0.
Evidence suggests means are different for full time & part-time staff.

d) allows us to assume sample means $\overline{F}$ and $\overline{P}$ are normally distributed.

e) sample variance = population variance.
(Test Statistic by definition uses population variance)

f) $2.53 < 2.5758$
Result is insignificant.
Accept H_0.
Insufficient evidence to suggest a difference in mean scores of full-time & part-time staff.

g) The training course has improved the awareness of part-time staff substantially as now there is an insignificant difference between full-time and part-time staff test scores. In (c) before the training, full-time staff were more aware as a group.